Derivation: Velocity of Approach Equals Velocity of Recession

Provided the collision is totally elastic, the velocity of approach equals the velocity of recession, regardless of the masses and initial velocities.

Initial Conditions:

- ☐ Mass 1 has mass m_1, velocity v_{1i}
- ☐ Mass 2 has mass m_2, velocity v_{2i}
- ☐ m_1 collides elastically with m_2
- ☐ After the collision, m_1 moves with velocity v_{1f} and m_2 with velocity v_{2f}

Conservation of Momentum:

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$ Equation (1)

Conservation of Energy:

$$KE_i = KE_f$$

$$\frac{1}{2} m_i v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_i v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$ Equation (2)

After re-arranging Equation (2) so that m_1 and m_2 are on opposite sides, factoring the masses, and finally expressing the difference in the velocities as the difference of two squares, we get:

$$\frac{1}{2} m_i v_{1i}^2 = \frac{1}{2} m_i v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_2 (v_{2f}^2 - v_{2i}^2) = m_1 (v_{1i}^2 - v_{1f}^2)$$

$$m_2 (v_{2f} - v_{2i})(v_{2f} + v_{2i}) = m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f})$$ Equation (3)

Now we re-arrange Equation (1) so that masses are on opposites sides:

$$m_2(v_{2f} - v_{2i}) = m_1(v_{1i} - v_{1f})$$ Equation (4)

Now we divide Equation (3) by Equation (4):

$$\frac{m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) = m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f})}{m_2(v_{2f} - v_{2i}) = m_1(v_{1i} - v_{1f})}$$

$$v_{2f} + v_{2i} = v_{1i} + v_{1f}$$ Equation (5)

By rearranging terms in Equation (5), we obtain the final form of the result.

$$v_{2f} - v_{1f} = -(v_{2i} - v_{1i})$$

Velocity of approach equals velocity of recession.