12 Projectile Motion
Sharp Shooter

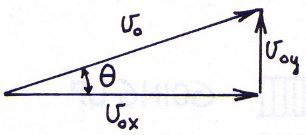
Purpose

To predict the range of a projectile fired at an angle to the horizontal.

Required Equipment and Supplies

ballistic pendulum apparatus and brick
carbon paper
meterstick
plumb bob
masking tape
protractor
graph paper
Apple II Series computer (optional)
"Sharp Shooter" simulation (optional)

Discussion



Recall from the lab "Bull's Eye" that the range of a projectile was predicted by measurements of the horizontal velocity and the height of a projectile. The initial motion of the projectile was purely horizontal, so the initial velocity, v_0, was equal to the horizontal component of the projectile's velocity throughout the entire trajectory.

In this lab you will first compute the muzzle velocity, v_0, of a projectile when fired horizontally by measuring the range R and vertical height h. You will then predict the range of the projectile when fired at a known angle θ. If the "Sharp Shooter" simulation is available, use it to check your calculations.

There are several ways to proceed. One way estimates the range by successive calculations; another involves breaking the problem up into two parts; and a third way uses the quadratic formula. Whatever method you choose, be conscious of two very important and fundamental concepts of projectile motion:

Concept 1

The range of a projectile is the product of its horizontal velocity, v_{0x}, and the time of flight, t.

$$R = v_{0x}t$$

$$= (v_0 \cos\theta)t$$

A graph of the horizontal component of the initial velocity, v_0, looks like Figure 12.1.

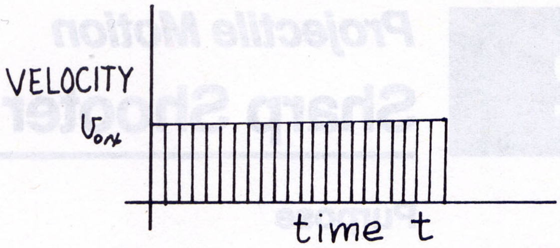


Figure 12.1

Concept 2
The time the projectile travels horizontally equals the time it travels vertically.

The total time, t, the projectile is in the air is the sum of the time it takes to reach its maximum height (time up), t_1, and the time the projectile falls (time down), t_2. That is,

$$t = t_1 + t_2$$

The maximum velocity of the projectile is its initial velocity, v_0. Its maximum vertical component of velocity, v_y, is v_{0y}, which continually decreases 9.8 m/s each second the projectile ascends (assuming negligible air drag). A graph of the vertical component of velocity, v_y, looks like Figure 12.2.

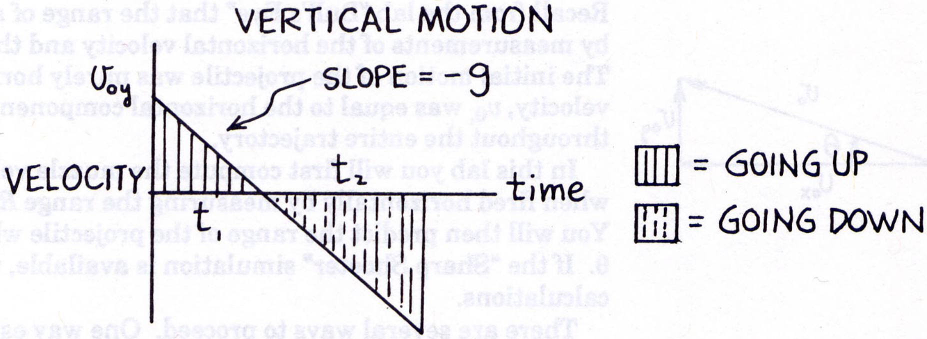


Figure 12.2

Method 1: Successive Approximation

Once the initial velocity of the projectile is known, its horizontal and vertical components can be calculated directly:

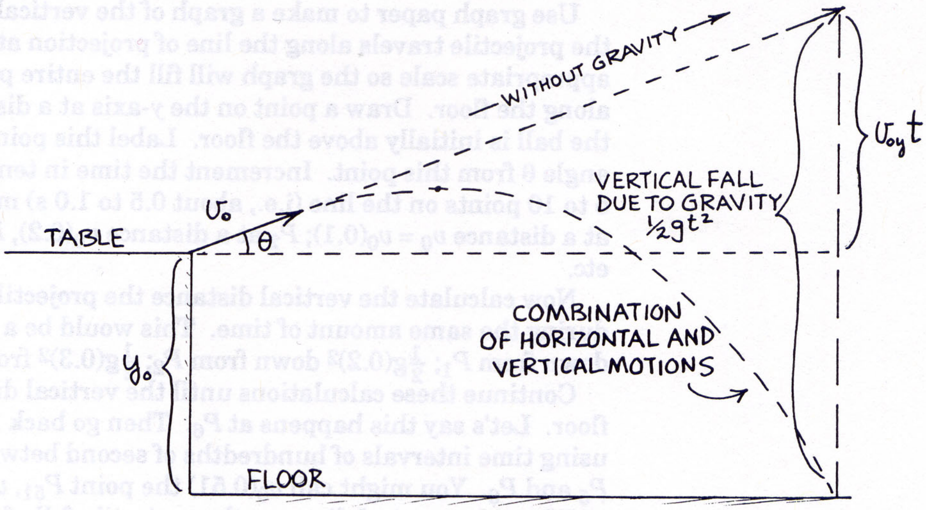
$$v_{0y} = v_0 \sin\theta$$

$$v_{0x} = v_0 \cos\theta$$

The vertical and horizontal position of the projectile at any time later *in the absence of gravity* would be

vertical height: $h = v_{0y}t$

horizontal range: $x = (v_0 \cos\theta)t$



However, since the earth accelerates all things downward at the same rate regardless of their horizontal motion, the actual vertical position of the projectile will be a distance $\frac{1}{2}gt^2$ lower than h.

$$y = h - \frac{1}{2}gt^2$$

$$y = v_{0y}t - \frac{1}{2}gt^2$$

These expressions for the height of the projectile, y, assume that the initial vertical position of the projectile was $y = 0$, in which case it will land on the floor at a height, $-y$. Alternatively, we can choose the initial height of the projectile to be a positive distance y_0 above the floor so that it lands on the floor at a vertical height, $y = 0$. The formula for the vertical height then becomes

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

When the projectile lands on the floor, y = 0, or

$$0 = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

The vertical height the projectile falls equals the vertical distance it would have risen plus the height it was above the floor when it was fired:

$$\frac{1}{2}gt^2 = y_0 + v_{0y}t$$

Since the final vertical position of the projectile below the horizon is measured, the only unknown in the above equation is time, t. Using a calculator, values of t can be incremented until the vertical distance the projectile falls, $\frac{1}{2}gt^2$, equals the vertical distance, $v_{0y}t$, it would have risen above its initial firing position, y_0.

Equivalently, the position of the projectile at any time later in the absence of gravity would be v_0t along the line of projection treating the initial height of the projectile above the floor as positive distance. The vertical distance the projectile falls due to gravity from this line at any time t is $\frac{1}{2}gt^2$. This technique involves estimating how far the projectile has to rise along the line of sight, v_0t, in order to fall a distance $\frac{1}{2}gt^2$ to the floor.

Use graph paper to make a graph of the vertical and horizontal distance the projectile travels along the line of projection at an angle θ. Choose an appropriate scale so the graph will fill the entire page. The x-axis will be along the floor. Draw a point on the y-axis at a distance equal to the height the ball is initially above the floor. Label this point P_0. Draw a line at an angle θ from this point. Increment the time in tenths of a second and make 5 to 10 points on the line (i.e., about 0.5 to 1.0 s) marking the first one, P_1, at a distance $v_0 = v_0(0.1)$; P_2 at a distance $v_0(0.2)$, P_3 at a distance $v_0(0.3)$, etc.

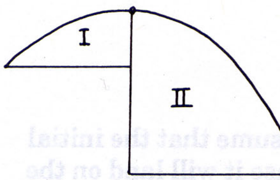
Now calculate the vertical distance the projectile falls from these points during the same amount of time. This would be a distance $\frac{1}{2}gt^2 = \frac{1}{2}g(0.1)^2$ down from P_1; $\frac{1}{2}g(0.2)^2$ down from P_2; $\frac{1}{2}g(0.3)^2$ from P_3, etc.

Continue these calculations until the vertical drop of the ball is *below* the floor. Let's say this happens at P_6. Then go back and repeat the process using time intervals of hundredths of second between the last two points, or P_5 and P_6. You might call $v_0(0.51)$ the point P_{51}, $v_0(0.52)$ the point P_{52}, etc.

When the vertical distance the projectile falls from the line of projection most nearly equals the distance to the floor, the horizontal range can be measured along the horizontal axis using the same distance scale as the vertical axis.

Method 2: Two-Stage Solution

This method involves breaking the trajectory into two parts:



Part I—from its initial height, y_0, to the maximum height of the projectile
Part II—from the maximum height until it lands

The horizontal range is the product of the horizontal component of the velocity and the total time the projectile is in flight:

$$R = v_{0x}t$$

The horizontal component of the initial velocity can be computed directly:

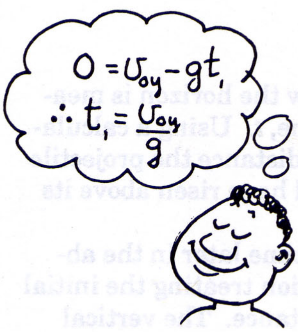
$$v_{0x} = v_0 \cos\theta$$

The time of flight, t, can be computed by adding the time it takes to reach the maximum height, t_1, (Part I) and adding it to the time, t_2, it takes to fall from the maximum height to the floor (Part II).

a) Calculate the horizontal and vertical components of the initial speed, v_{0x} and v_{0y}:

v_{0x} = _______

v_{0y} = _______



b) Calculate the time it takes to reach the maximum height, t_1:

t_1 = _______

c) Calculate the maximum height, y_1:

y_1 =

d) Calculate how long it takes the projectile to reach the ground from the maximum height, t_2:

$t_2 =$ ________

e) Calculate the total time the projectile is in flight, $t = t_1 + t_2$:

$t =$ ________

f) Calculate the range, $R = v_{0x}t$

$R =$ __________

Method 3: Quadratic Formula

The vertical position of the projectile, y, at any time, t, after it is fired is:

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

Since the only unknown in the above equation is the time, t, the quadratic formula can be used to solve for t. First, rearrange the equation so that it is in standard form:

$$\frac{1}{2}gt^2 - v_{0y}t - y_0 = 0$$

The coefficients are then:

$$a = \frac{1}{2}g$$
$$b = -v_{0y}$$
$$c = -y_0$$

The solutions for t are:

$$t = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{v_{0y} + \sqrt{v_{0y}^2 + 4\frac{g}{2}y_0}}{g} = \frac{v_{0y} + \sqrt{v_{0y}^2 + 2gy_0}}{g}$$

and

$$t = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{v_{0y} - \sqrt{v_{0y}^2 + 2gy_0}}{g}$$

One solution for t will be positive, the other will be negative. The positive solution is the time the projectile travels from its initial position to its final position. The negative solution is the time it would take if it were fired at a position $-y$ with an initial speed of v_{0y} and ended up at $y = 0$.

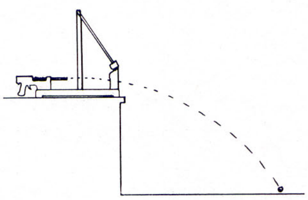
The range is the product of the horizontal component of the velocity and the total time the projectile is in flight:

$$R = v_{0x}t$$

Data Table 12.1

TRIAL	RANGE	% DIFFERENCE
AVERAGE RANGE =		
AVERAGE % DIFFERENCE =		

Procedure

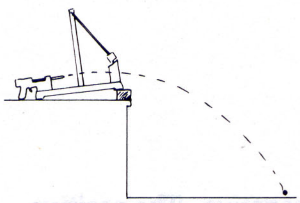


Step 1. Position a ballistic pendulum apparatus on the lab table. Remove the pendulum from the apparatus (or prop it up) so it can be used to fire a ball from the lab table. Arrange the apparatus so it is level on the table. Secure the apparatus to the lab table with a clamp, if necessary, to preclude motion of the apparatus when the ball is fired. Use care and precaution when firing. Fire the steel ball several times (5–10 times) until you get consistent results. Measure the range of the projectile and the vertical distance it falls to the floor to compute the muzzle velocity, v_0. Calculate the average range of the projectile. Calculate the percentage difference between the average value and each measurement of the range. Then calculate the average percentage difference. Record your results in Data Table 12.1.

Step 2. Prop a brick under the front feet of the ballistic pendulum apparatus so that it projects the ball at an angle to the lab table. Measure the angle of projection, θ. Measure the vertical height the ball is now from the floor.

θ = _______

h = _______



Step 3. Using any of the methods outlined in the discussion, predict the range of the projectile in this elevated position. Check your calculations with your instructor and with those of the computer, if available. Show your calculations.

R = _______

Compare your prediction to that of the "Sharp Shooter" simulation program. If they do not agree within 2%, go back and check your calculations.

computer prediction for R = ________

Step 4. Use a plumb bob to mark the position on the floor directly below where the projectile is fired and mark it with a piece of masking tape. Measure a distance, R, from this position to where you predict the ball will land. Place a carbon paper underneath a piece of filler paper so the ball will leave a mark when it lands.

Step 5. Fire the ball and record where it lands. Measure the actual range. Repeat several times and calculate the average range.

average range = ________

Analysis

1. Calculate the percentage difference between your prediction and the average range.

percentage difference = ________

2. Is the percentage difference of the average range greater or less than the percentage difference you calculated when the projectile was fired horizontally?

3. What sources of error might account for any differences in the predicted range and the actual range of the projectile? Account for any discrepancy.