The Anatomy of a Pitch: Doing Physics with PITCHf/x Data

David Kagan
Department of Physics
California State University, Chico
Chico, CA 95929-0202
dkagan@csuchico.edu

On August 7th, 2007 Barry Bonds of the San Francisco Giants was at bat waiting for a 3-2 pitch from Mike Bacsik of the Washington Nationals. The ball left the pitcher's hand at 84.7[1] mph and arrived at home plate traveling 77.2 mph. It was within 0.2 inches of the center of home plate and 3.213 feet above the ground when Bonds swung and hit his 756th home run making him the all-time leader for homers in a career[2]. Thanks to a company called Sportsvision[3] and Major League Baseball[4] (www.mlb.com) you can get kinematic data on any pitch thrown. Just think of the interesting and realistic physics problems you can generate from such a data set!

Video technology to track pitches started to be installed in every major league ballpark in 2007. The system is called PITCHf/x[5]. The fabulous thing for you and your students is that the data is provided at no cost by Major League Baseball (although it is copy righted). Alan Nathan has a good web site (webusers.npl.uiuc.edu/~a-nathan/pob/tracking.htm) to describe how to download the data and what the quantities actually mean[6]. The easiest way to get the data is from Dan Brooks's web site PITCHf/x Tool[7].

While you or your students might find another pitch more interesting, the pitch described above will be used here to illustrate some of the physics you can do. Table 1 contains the information about the pitch Bonds hit[8]. Figure 1 illustrates the coordinate system used. The origin is at the back point of home plate on the ground. The x-axis points to the catcher's right when he is facing the pitcher. The y-axis points directly toward the pitcher so the y-component of the pitched ball's velocity is always negative. The z-axis is oriented upward.

The PITCHf/x software calculates the trajectory of each pitch using input from video cameras dedicated to the system. It uses the trajectory data to perform a least squares fit to the initial position, initial velocity, and acceleration along each axis. Note that the acceleration is assumed constant over the entire flight of the pitch. This assumption leads to equations that claim to reproduce the actual trajectory to within an inch or two. For the pitch we are going to look at these values are in table 1 (items 14 – 22). Keep in mind that this data is processed in real time during the game. Information on each pitch is available in a fraction of a second.

Let's start by finding the initial speed of the ball (at y=50.0ft). In 3-dimensions the initial speed is the magnitude of the initial velocity vector. Since the components are listed in the table (items 17, 18, and19) we take the square root of the sum of their squares,

$$v_o = \sqrt{v_{ox}^2 + v_{oy}^2 + v_{oz}^2}$$

$$v_o = \sqrt{(-6.791)^2 + (-123.055)^2 + (-5.721)^2}$$

$$v_o = 123.375\, ft/s = 84.1\, mph^{9}.$$

This agrees precisely with the PITCHf/x value listed in table 1 (item 6).

We can find the components of the final velocity of the pitch when it reaches the front of home plate (y=1.417ft) by using the kinematic equations. Since we know the initial and final y-values we can get the y component of the velocity using the kinematic equation,

$$v_y^2 = v_{oy}^2 + 2a_y(y - y_o) \Rightarrow v_y = -\sqrt{v_{oy}^2 + 2a_y(y - y_o)}$$

Note that we want the negative value of the root to agree with the coordinate system. Plugging in the values,

$$v_y = -\sqrt{(-123.055)^2 + 2(25.802)(1.417 - 50.00)} = -112.408\,ft/s$$

The time of flight must be found to get the other velocity components. Using another kinematic equation,

$$v_y = v_{oy} + a_y t \Rightarrow t = \frac{v_y - v_{oy}}{a_y} = \frac{-112.408 - (-123.055)}{25.802} = 0.4127s.$$

Having the time of flight and using kinematic equations for the other two axes,

$$v_x = v_{ox} + a_x t = -6.791 + (13.233)(0.4127) = -1.330\,ft/s$$
$$v_z = v_{oz} + a_z t = -5.721 + (-17.540)(0.4127) = -12.960\,ft/s.$$

The final velocity vector is,

$$\vec{v} = (-1.330\,ft/s)\hat{i} + (-112.408\,ft/s)\hat{j} + (-12.960\,ft/s)\hat{k}.$$

Calculating the final speed,

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{(-1.330)^2 + (-112.408)^2 + (-12.960)^2} = 113.160\,ft/s = 77.2mph,$$

again in agreement with table 1 (item 7).

At typical batter doesn't get a sense of the motion of the pitch until the ball is about 40ft away from home plate. Let's find the time it takes the ball to get to y=40ft and the x and z components of the position and velocity when it gets there. Using the kinematic equation,

$$y = y_o + v_{oy}t + \tfrac{1}{2}a_y t^2 \Rightarrow t_{40} = \frac{-v_{oy} \pm \sqrt{v_{oy}^2 - 2a_y(y_o - y)}}{a_y}.$$

We will need to use minus sign,

$$t_{40} = \frac{-(-123.055) - \sqrt{(-123.055)^2 - 2(25.802)(50 - 40)}}{(25.802)} = 0.08197s.$$

The x position and velocity can now be found,

$$x_{40} = x_o + v_{ox}t_{40} + \tfrac{1}{2}a_x t_{40}^2 = 1.664 + (-6.791)(0.08197) + \tfrac{1}{2}(13.233)(0.08197)^2 = 1.152\,ft$$
$$v_{x40} = v_{ox} + a_x t_{40} = -6.791 + (13.233)(0.08197) = -5.706\,ft/s,$$

as can the z position and velocity,

$$z_{40} = z_o + v_{oz}t_{40} + \tfrac{1}{2}a_z t_{40}^2 = 6.597 + (-5.721)(0.08197) + \tfrac{1}{2}(-17.540)(0.08197)^2 = 6.069\,ft$$
$$v_{z40} = v_{oz} + a_z t_{40} = -5.721 + (-17.540)(0.08197) = -7.159\,ft/s.$$

Now that the batter has a sense of the position and velocity of the ball he can begin to plan his swing. The batter has plenty of experience dealing with the force that the air exerts on the ball due to the drag of air resistance. Therefore, the batter might expect the ball to arrive at home plate at the usual rate determined by the actual speed he can estimate at y=40ft and typical air resistance. The time of flight from y=40ft can be found from by subtracting the total time from the time to get to y=40ft,

$$t_h = t - t_{40} = 0.4127 - 0.08197 = 0.3307s$$

The batter has a much more difficult time estimating the spin the pitcher put on the ball. The spin causes the air to exert additional forces on the ball due to the Magnus Effect. Suppose the air had no effect on the motion of the ball in the x and z directions starting at the point y=40ft. Find the x and z positions of the ball when it gets to the front of home plate. Along the x-direction there would be no acceleration,

$$x_{noair} = x_{40} + v_{x40}t_h + \tfrac{1}{2}a_x t_h^2 \Rightarrow x_{noair} = 1.152 + (-5.706)(0.3307) = -0.735\,ft.$$

Along z there would only be gravity,

$$z_{noair} = z_{40} + v_{z40}t_h + \tfrac{1}{2}a_z t_h^2 \Rightarrow z_{noair} = 6.069 + (-7.159)(0.3307) + \tfrac{1}{2}(-32.174)(0.3307)^2 = 1.942\,ft.$$

This is where a non-experience batter might think the pitch will be if it had no spin.

One way batters describe the effect of spin is called the "break." One way to analytically define the break is the difference between where the ball actually arrives and where is would have arrived without any spin. The actual x and z positions are in table 1 (items 12 and 13). So, this definition of break can now be calculated for the x and z directions.

$$x_{break} = x - x_{noair} = -0.012 - (-0.735) = 0.723\,ft = 8.68\,in.$$

$$z_{break} = z - z_{noair} = 2.743 - 1.942 = 0.801\,ft = 9.61\,in.$$

These values are very close to the pfx_x and pfx_z values in table 1 (items 10 and 11). This method of calculating the "break" of the pitch is somewhat arbitrary. Other methods are possible and the last three pieces of data in table 1 (items 23, 24, and 25) refer to a different method[10].

Let's look at the forces involved in the motion of a major league pitch. Given the weight of a baseball is 0.320lbs, we can find the x, y, and z components of the force exerted on the ball by the air during its flight. Since the components of the acceleration are given in table 1 (items 20, 21, and 22), we can use Newton's Second Law along each direction. Along x and y the only force is due to the air,

$$F_x = ma_x = mg\left(\frac{a_x}{g}\right) = (0.320)\left(\frac{13.233}{32.174}\right) = 0.132\,lbs$$

$$F_y = ma_y = mg\left(\frac{a_y}{g}\right) = (0.320)\left(\frac{25.802}{32.174}\right) = 0.257\,lbs.$$

Along z gravity is also in play,

$$F_z - mg = ma_z \Rightarrow F_z = mg + mg\left(\frac{a_z}{g}\right) = mg\left(1 + \frac{a_z}{g}\right) = (0.320)\left(1 + \frac{-22.232}{32.174}\right) = 0.146\,lbs$$

The magnitude of the force caused by the air is,

$$F_{air} = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(0.132)^2 + (0.257)^2 + (0.146)^2} = 0.324\,lbs$$

The largest force is in the y-direction and is predominantly the drag of air resistance. The upward force of lift on the spinning ball in the z-direction is mostly created by the backspin about the x-axis while the force in the x-direction is from the sidespin about the z-axis. These are lifts that are related to the Magnus Effect. It is amazing to realize the major league pitchers can spin a ball fast enough and give it sufficient velocity that the total force caused by the air is about the same as the weight of the ball.

The physics of pitching has been described many times before.[11] However, with the PITCHf/x data set intricate studies of the flight of real pitches in actual game situations can be thoroughly examined. Alan Nathan's "The Physics of Baseball[12]" web site is a great place to start if you want to get a sense of what intrepid souls are doing with PITCHf/x data. Now, when your students ask a question about the flight of baseballs, you can direct them to real life data from an actual pitch from their favorite pitch or a pitch to their favorite hitter.

No.	Quantity	Value	Units	Description
1	des	In play, run(s)		A comment on the action resulting from the pitch.
2	type	X		B=ball, S=strike, X=in play
3	id	371		Code indicating pitch number
4	x=	112.45	pixels	x-pixel at home plate
5	y=	131.24	pixels	z-pixel at home plate (yes, it is z)
6	start_speed	84.1	mph	Speed at y0=50ft
7	end_speed	77.2	mph	Speed at the front of home plate y=1.417ft
8	sz_top	3.836	ft	The z-value of the top of the strike zone as estimated by a technician
9	sz_bot	1.79	ft	The z-value of the bottom of the strike zone as estimated by a technician
10	pfx_x	8.68	in	A measure of the “break” of the pitch in the x-direction.
11	pfx_z	9.55	in	A measure of the “break” of the pitch in the y-direction.
12	px	-0.012	ft	Measured x-value of position at the front of home plate (y=1.417ft)
13	pz	2.743	ft	Measured z-value of position at the front of home plate (y=1.417ft)
14	x0	1.664	ft	Least squares fit (LSF) value for the x-position at y=50ft
15	y0	50.0	ft	Arbitrary fixed initial y-value
16	z0	6.597	ft	LSF value for the z-position at y=50ft
17	vx0	-6.791	ft/s	LSF value for the x-velocity at y=50ft
18	vy0	-123.055	ft/s	LSF value for the y-velocity at y=50ft
19	vz0	-5.721	ft/s	LSF value for the z-velocity at y=50ft
20	ax	13.233	ft/s/s	LSF value for the x-acceleration at y=50ft assumed constant throughout the pitch.
21	ay	25.802	ft/s/s	LSF value for the y-acceleration at y=50ft assumed constant throughout the pitch.
22	az	-17.540	ft/s/s	LSF value for the z-acceleration at y=50ft assumed constant throughout the pitch.
23	break_y	25.2	ft	Another measure of the “break.” See Nathan’s website for an explanation.
24	break_angle	-32.1	deg	Another measure of the “break.” See Nathan’s website for an explanation.
25	break_length	5.9	in	Another measure of the “break.” See Nathan’s website for an explanation.

Table 1: The names of each quantity, its value, its units, and a brief description. For more complete information, see Alan Nathan’s web site (webusers.npl.uiuc.edu/~a-nathan/pob/tracking.htm).

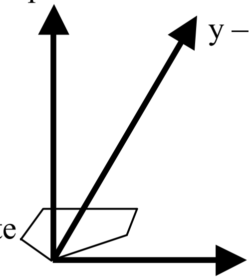

Figure 1: The coordinate system has its origin is at the back point of home plate on the ground. The x-axis points to the catcher's right. The y-axis is toward the pitcher. The z-axis is oriented upward.

---

[1] The author makes no apology for the use of English units throughout this paper. After all, they are the traditional units of the National Pastime!
[2] An amazingly sharp 30-frames/second video of Bonds hitting the ball can be found at http://www.usatoday.com/sports/graphics/bonds-756/flash.htm. Video of the entire at bat as well as just the last pitch can be found by searching Major League Baseball's site mlb.com.
[3] www.sportvision.com is their home page
[4] www.mlb.com is their home page. You may want to check out their Gameday feature that uses this data at http://mlb.mlb.com/mlb/gameday/.
[5] Sportvision's description is at http://www.sportvision.com/main_frames/products/pitchfx.htm.
[6] webusers.npl.uiuc.edu/~a-nathan/pob/tracking.htm
[7] www.brooksbaseball.net/pfx/
[8] Ironically, just before this fateful pitch, the pitcher requested a new baseball. He tossed home to the catcher and this event fooled the software into thinking it was an actual pitch. The data from this toss is what you will get if you use PITCHf/x Tool. MLB has put the corrected data on its server and that is where the author collected it.
[9] The author has made little effort to track significant figures because the data from PITCHf/x has little regard for them. Just keep in mind that trajectory data is good to about one-to-two inches.
[10] See Nathan's web site, webusers.npl.uiuc.edu/~a-nathan/pob/tracking.htm, for an explanation.
[11] several physics of pitching references
[12] webusers.npl.uiuc.edu/~a-nathan/pob/index.html